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Properties of Abelian Monopoles in $SU(2)$ Lattice Gluodynamics *

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Abstract: We discuss some properties of abelian monopoles in the Maximal Abelian projection of the $SU(2)$ lattice gluodynamics. We show that in the maximal abelian projection abelian monopoles carry fluctuating electric charge and that the monopole currents are correlated with the magnetic and the electric parts of the $SU(2)$ action density.

1 Introduction

Abelian monopoles play a key role in the dual superconductor mechanism of confinement [1] in non-abelian gauge theories. Abelian monopoles appear after the so called abelian projection [2]. Condensation of abelian monopoles gives rise to the formation of an electric flux tube between the test quark and antiquark. Due to a non-zero string tension the quark and the antiquark are confined by a linear potential. There are many numerical facts [3] which show that the abelian monopoles in the Maximal Abelian (MaA) projection are responsible for the confinement. The monopole condensation in the confinement phase of gluodynamics has been established by the investigation of various monopole creation operators [4] in the MaA projection [5]. The $SU(2)$ string tension is well described by the contribution of the abelian monopole currents [6]; these currents satisfy the London equation for a superconductor [7].

Below we discuss several recently found properties of the abelian monopole currents. In Section 2 we show that in the vacuum of the $SU(2)$ lattice gluodynamics the abelian monopoles currents are correlated with the electric currents. In Section 3 we show that the abelian monopoles are locally correlated with electric and magnetic parts of the $SU(2)$ action density. All numerical calculations are performed in the MaA projection.

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2 Abelian Monopoles Carry Electric Charge

Consider a (anti-) self–dual configuration of the $SU(2)$ gauge field:

$$F_{\mu\nu}(A) = \pm\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(A) \equiv \pm\tilde{F}_{\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu}(A) = \partial_{[\mu,}A_{\nu]} + i[A_\mu, A_\nu]$. In the MaA projection the commutator term $\mathrm{Tr}(\sigma^3[A_\mu, A_\nu])$ of the field strength tensor $F^3_{\mu\nu}$ is suppressed, since the MaA projection is defined [5] by the minimization of the functional $R[A] = \int d^4x[(A^1_\mu)^2 + (A^2_\mu)^2]$ over the gauge transformations. Thus, in the said projection, the fields $A_\mu(x)$ are as close to abelian (diagonal) fields as possible. Therefore, in the MaA projection eq.(1) yields [8]: $f_{\mu\nu}(A) = \partial_\mu A^3_\nu - \partial_\nu A^3_\nu \approx \pm\tilde{f}_{\mu\nu}(A)$. Thus, the abelian monopole currents must be accompanied by the electric currents: $J^e_\mu = \partial_\nu f_{\mu\nu}(A) \approx \pm\partial_\nu\tilde{f}_{\mu\nu}(A) = \pm J^m_\mu$. Therefore, in the MaA projection the abelian monopoles are dyons for (anti) self-dual $SU(2)$ field configurations [8]. Below we show that in the real (not cooled) vacuum of lattice gluodynamics the abelian monopole currents are correlated with the electric currents [9].

In order to study the relation of electric and magnetic currents, we have to calculate connected correlators of these currents. The simplest correlator $\ll J^m_\mu J^e_\mu \gg \equiv\ < J^m_\mu J^e_\mu > - < J^m_\mu >< J^e_\mu >$ is zero, since $< J^m_\mu J^e_\mu >= 0$ due to the opposite parities of the operators J^m and J^e, and $< J^{m,e}_\mu >= 0$ due to the Lorentz invariance. The simplest non–trivial (normalized) correlator is

$$\bar{G} = \frac{1}{\rho^e\rho^m}\ < J^m_\mu(y)J^e_\mu(y)q(y) >\,, \tag{2}$$

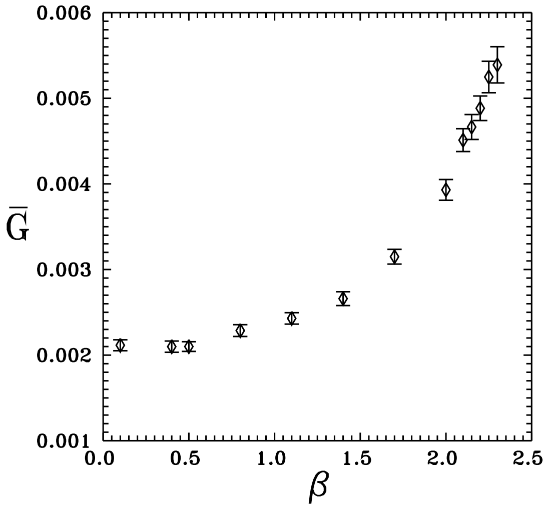
where $q(x)$ is the sign of the topological charge density at the point x and $\rho_{m,e} = \sum_l < |J^{m,e}_l| > /(4V)$ are the densities of the magnetic and the electric charges, V is the lattice volume (total number of sites).

We perform a numerical calculation of the correlator (2) in the $SU(2)$ lattice gauge theory on the 8^4 lattice with periodic boundary conditions. We use 100 statistically independent gauge field configurations for each value of β.

The dependence of the correlator $\bar{G}$ on β is shown in Fig. 1(a). This correlator is positive for all values of β. Therefore, the abelian monopoles in the MaA projection carry an electric charge, too. According to definition (2), the sign of the electric charge of the monopole coincides with the product of the magnetic charge and the topological charge. Thus, in the gluodynamic vacuum the abelian monopoles become abelian dyons due to a non-trivial topological structure of the vacuum gauge fields.

3 Abelian Monopole Currents are Correlated with $SU(2)$ Action Density

Abelian monopoles appear as singularities in the gauge transformations [2, 3]. On the other hand, the monopole currents reproduce the $SU(2)$ string tension [6]. Thus,



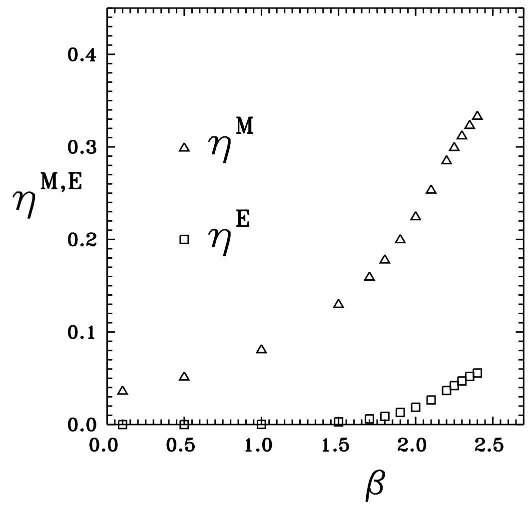


Figure 1: (a) The dependence of the correlator $\bar{G}$ on β; (b) The relative excess of the magnetic (circles, from Refs. [11]) and the electric (boxes) action density near the monopole current. The data are extrapolated to the infinite lattice size.

monopoles are likely to be related to some physical objects. A physical object is something which carries action. Below we study the local correlations of the abelian monopoles with the density of the magnetic and the electric parts of the $SU(2)$ action (the global correlation was found in Ref. [10]). We show that the monopoles are physical objects but it does not mean that these have to propagate in the Minkowski space; a chain of instantons can produce a similar effect: an enhancement of the action density along a line in Euclidean space. The simplest quantities which can show this correlation are the relative excess of the magnetic and the electric action densities $\eta^{M,E} = (S^{M,E}_m - S)/S$ in the region near the monopole current. Here S is the expectation value of the lattice plaquette action, $S_P =< (1 - \frac{1}{2} Tr\, U_P) >$. The quantities $S^{M,E}_m$ are, respectively, the magnetic and the electric parts of the $SU(2)$ action density, which are calculated on plaquettes closest to the monopole current.

In the continuum notation, the quantities $S^{M,E}_m$ have the following form:

$$S^M_m = \frac{1}{2} < \mathrm{Tr}(n_\mu(x)\, \tilde{F}_{\mu\nu}(x))^2 > , \quad S^E_m = \frac{1}{2} < \mathrm{Tr}(n_\mu(x)\, F_{\mu\nu}(x))^2 > , \tag{3}$$

$n_\mu(x)$ is the unit vector in the direction of the current: $n_\mu(x) = j_\mu(x)/|j_\mu(x)|$, if $j_\mu(x) \neq 0$, and $n_\mu(x) = 0$ if $j_\mu(x) = 0$. It is easy to see that for a static monopole ($j_0 \neq 0;\ j_i = 0, i = 1,2,3$) S^M_m (resp., S^E_m) corresponds to the chromomagnetic action density $(B^a_i)^2$ (resp., chromoelectric action density $(E^a_i)^2$) at the monopole current.

We calculate the quantities η^M and η^E on symmetric lattices L^4 of different lattice size $L = 8, 10, 12, 16, 20, 24, 30$ with periodic boundary conditions. In Fig. 1(b) we show the quantities $\eta^{M,E}$ extrapolated to the infinite lattice size, $(L \to \infty)$ $vs.$ β. The

monopole currents are calculated in the MaA projection. In Fig. 1(b) the statistical errors are smaller than the size of the symbols. It is clearly seen that the abelian monopoles are correlated with both the magnetic and the electric parts of the $SU(2)$ action density. Note that the correlation of the monopole charge with the magnetic action density is larger than the correlation with the electric part of the $SU(2)$ action.

Conclusion and Acknowledgments

Our results show that the abelian monopoles in the MaA projection of the $SU(2)$ gluodynamics *i)* have a fluctuating electric charge; *ii)* carry the $SU(2)$ action.

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